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ABSTRACT

Two task were developed to assess second and fourth grade girls' ability to generate quantitative inferences and estimate answers to math word problems. The generation of quantitative inferences is thought to constitute problem representation, and the ability to estimate answers to word problems is facilitated by the presence of an accurate problem representation. It was hypothesized that individual differences in these abilities would be related to mathematics achievement. Twenty second and 20 fourth graders were individually administered simple computation sums and two tasks designed to measure the ability to generate and use quantitative inferences. The Mean Retention Task (MRT) required children to generate a quantitative relational inference and retain it in memory. The Math Inference Task (MIT) consisted of word problems with several alternate answers. The child was required to generate a quantitative inference and use it to choose only those answers that were possibly correct. Fourth graders did better than second graders on computation and the MIT, but there was no difference between grades on the MRT. Performance on the MIT and the MRT was significantly correlated with scores on standardized math achievement tests in fourth grade only. Computation scores were a factor in determining mathematical problem solving ability in second grade only. Finally, MIT and MRT were significantly correlated with each other in both grades. (Author)

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Individual Differences in Quantitative Inferencing Ability and Mathematical Ability.

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ABSTRACT

Two tasks were developed to assess second and fourth grade girls' ability to generate quantitative inferences and estimate the answers to math word problems. The generation of quantitative inferences is thought to constitute problem representation, and the ability to estimate answers to word problems is facilitated by the presence of an accurate problem representation. It was hypothesised that individual differences in these abilities would be related to mathematics achievement. 20 second and 20 fourth graders were individually administered simple computation sums and two tasks designed to measure the ability to generate and use quantitative inferences. The Meaning Retention Task (MRT) required children to generate a quantitative relational inference and retain it in memory. The Math Inference Task (MIT) consisted of word problems with several alternate answers. The child was required to generate a quantitative inference and use it to choose only those answers that were possibly correct.

Fourth graders did better than second graders on computation and the MIT, but there was no difference between grades on the MRT. Performance on the MIT and the MRT was significantly correlated with scores on standardized math achievement tests in fourth grade only. Computation scores were a factor in determining mathematical problem solving ability in second grade only. Finally, MIT and MRT were significantly correlated with each other in both grades.

INTRODUCTION

The purpose of this research was to investigate the antecedents and development of successful word problem solution and general mathematical abilities. Specifically, we were concerned with the development of two abilities which have been hypothesised as being important in the solution of word problems - the ability (1) to make quantitative inferences and (2) to use this inference to estimate the answer to a specific word problem. Each of these was in turn hypothesised to be dependent on an accurate mental representation of the problem. Thus, we attempted to assess the existence and development of the accurate representation of mathematical word problems, and the relationship between the accuracy of information in this representation and performance on standardised mathematical tests.

A problem representation, according to Riley, Greeno and Heller (1983) is what results from reading and comprehending text concerning the problem. It involves building a coherent representation of relationships among the individual elements contained in the problem. Kintsch and Greeno (1985) have proposed a problem solving model in which the problem representation, which they call the situation model, is constructed independently of the way in which the text of the word problem is presented. The situation model contains inferences based on the text, as well as other relevant domainrelated knowledge. These inferences are made in accordance with task demands that the reader expects

to encounter (i.e., in mathematical word problems, they guide the choice of operations for the right solution).

Such issues of translation from literal to situational representation have been studied elsewhere in cognitive research. In 1980, Johnson-Laird described a mental model as being constructed from the major propositions in the text, and including implicit inferences not mentioned in the text. These mental models were used to describe humans' comprehension of spatial descriptions, and their construction was found to be associated with better recall of the text material. Johnson-Laird also found that memory for the information in a mental model was better than for the verbatim content of the text presented. He theorised that the mental model is the result of further processing on the verbatim propositions in the text, and therefore has greater probability of being recalled than the verbatim propositions. The results of his experiments have provided evidence to support this. Mental models have been used in describing individuals' comprehension of rather advanced knowledge domains - mechanics, electricity, the use of hand calculators (e.g. William, Hollan and Stevens, 1983). They have also been used to describe spatial and temporal domains (Wagener and Wender, 1985). In reading this literature we noticed many similarities between the mental models discussed here and the situation models described earlier in the mathematical literature.

It was our contention that, in order for young children to

solve word problems they would have to learn how or develop the ability to construct the correct "mathematically relevant" mental model. Specifically, we believed that the presence of quantitative relational inferences (more and less) would be associated with enhanced performance on math word problems. In addition, we believed we would find individual differences in the ability to build such mental models, that would be correlated with mathematical ability. As far as we know, this combination of concerns has not been previously addressed.

Method

20 second and 20 fourth grade girls were tested individually on two tasks designed to assess the abilities mentioned above, and a basic computation task involving addition and subtraction only. We also acquired standardized scores on a math achievement test from the school records. The two tasks are described below with examples.

Meaning Retention Task (MRT)

The task tested the hypothesis that good problem solvers, on reading a word problem, generate quantitative inferences, thus forming a mental model. Following Johnson-Laird's (1980) findings, we expected that probability of recall would be higher for these inferences than for verbatim text sentences, after a distractor task. An example of the task is given below.

"The zoo has monkeys and parrots. There are less parrots than monkeys. There are 56 parrots."

This is followed by a distractor task, and then a sentence

recognition task:

"Which of these sentences did you read just now?

1. There are more parrots than monkeys.
2. There are less monkeys than parrots.
3. There are 56 monkeys.
4. There are more monkeys than parrots."

Successful performance on this task was the 'misrecognition' of the fourth sentence in the example above.

Mathematics Inference Task (MIT)

This task tested the ability to generate and use a quantitative inference to estimate the answer to a word problem. An example is given below:

"Mary and John are friends. Mary has more cookies than John. She has 12 cookies. Which of these sentences could be true?

1. John has 11 cookies
2. John has 20 cookies
3. John has 5 cookies
4. John has 13 cookies
5. John has 12 cookies."

One point was given for each legitimate alternative picked by the child (options 1 and 3 above).

Results

The standardised scores ranged from the 1st to the 99th percentile. There was no significant difference between percentile scores in the two grades.

Fourth graders performed significantly better than second graders on the MIT, but not on the MRT. Performance on the two tasks was significantly correlated with math achievement scores in fourth grade only, together accounting for 39% of the variability. In second grade, MIT and MRT did not contribute significantly to the variance in achievement scores, but performance on the basic computation task did. Thus, MRT and MIT appear to increase in importance from second to fourth grade. In addition, MRT seems to be as difficult in fourth as in second grade.

We did find, as expected, that MIT and MRT were significantly related in both second and fourth grades. The MRT requires that one generate a quantitative inference, and the MIT requires that having generated it, one use it. Consistent with this idea, regression analyses in the fourth grade showed that although the two measures account for overlapping variance in math achievement scores, MIT accounts for a significant portion of the variance over and above this overlapping contribution.

We feel that the results of this study indicate the importance of mental representations of word problems as prerequisite to their successful solution, atleast in fourth graders. Specifically, it appears that the information that is most useful in quantitative mental models is that regarding the relations among the key elements in the problem. It appears that many textbooks encourage the 'keyword' approach and do not facilitate deeper understanding of the problem situation. Our research shows

that such an approach could be detrimental to the child's performance in mathematical problem solving, since it ignores the need for any kind of problem representation.

Table 1. Mean Performance Scores on all Variables for Second and Fourth Graders

Variable	Mean	S.D.	Range
	Grade 2 (n = 20)		
Math Achievement[1]	69.35	28.83	1 - 98
Computation	8.95	2.89	0 - 12
Math Inference Task (MIT)	12.60	4.28	6 - 20
Meaning Retention Task (MRT)	6.10	2.40	1 - 10
	Grade 4 (n = 20)		
Math Achievement[1]	72.50	20.80	27 - 98
Computation	11.15	1.50	6 - 12
Math Inference Task (MIT)	17.05	4.38	5 - 20
Meaning Retention (MRT)	6.60	2.58	0 - 10

[1]Percentile scores on ITBS_Mathematics.

Table 2. Intercorrelations Among Variables for Second and Fourth Grades.

Variable	2	3	4
	Grade 2 (n = 20)		
1. Math Achievement	.74**	.26	.28
2. Computation	—	.41	.44
3. Math Inference Task		—	.64**
4. Meaning Retention Task			—
	Grade 4 (n = 20)		
1. Math Achievement	.20	.62**	.49*
2. Computation	—	-.18	.07
3. Math Inference Task		—	.70**
4. Meaning Retention Task			—

*p<.05, **p<.01

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